

Outbursts of Young Stellar Objects

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ABSTRACT

We argue that the outbursts of the FU Orionis stars occur on timescales which are much longer than expected from the standard disc instability model with $\alpha_c \gtrsim 10^{-3}$. The outburst, recurrence, and rise times are consistent with the idea that the accretion disc in these objects is truncated at a radius $R_i \sim 40 R_\odot$. In agreement with a number of previous authors we suggest that the inner regions of the accretion discs in FU Ori objects are evacuated by the action of a magnetic propeller anchored on the central star. We develop an analytic solution for the steady state structure of an accretion disc in the presence of a central magnetic torque, and present numerical calculations to follow its time evolution. These calculations confirm that a recurrence time that is consistent with observations can be obtained by selecting appropriate values for viscosity and magnetic field strength.

Key words: accretion, accretion discs – stars: FU Orionis – stars: T Tauri – stars: magnetic fields – stars: pre-main-sequence

1 INTRODUCTION

The young stellar objects (YSOs) are stars which have not yet completed the process of star formation. They frequently retain some portion of their proto-stellar discs, from which mass accretion is thought to take place. The T Tauri stars are YSOs with masses $\lesssim 2 M_\odot$, and accretion rates in the region $\sim 10^{-7} - 10^{-8} M_\odot \text{yr}^{-1}$, e.g. Hartmann et al. (1998). T Tau stars are divided into two subclasses, Classical T Tau stars (CTTs) and Weak Line T Taus (WTTs). CTTs have been shown to possess discs by the observation of double peaked absorption lines, e.g. Hartmann & Kenyon (1985), and from the presence of an infra-red excess (Adams, Lada & Shu 1987; Kenyon & Hartmann 1987). CTTs have also been observed to produce extended emission (Beckwith et al. 1984) which is consistent with the existence of an accretion disc. WTTs are a faster spinning (Bouvier et al. 1995) type of YSO with weak absorption lines and a less pronounced infra-red excess (Hartmann 1998). The presence of a disc is less certain in these systems.

Some YSOs have been observed to undergo outbursts, during which they increase in brightness by up to two orders of magnitude on a timescale $t_{\text{rise}} \sim 1$ yr. These objects, termed FU Orionis stars, are otherwise very similar to CTTs. Therefore it is generally assumed that FU Ori stars are members of the same population as T Tau

stars, e.g. Hartmann & Kenyon (1985). The outburst duration has been estimated as $t_{\text{ob}} \gtrsim 50$ yr (Hartmann 1998), from the decay times of the outburst light curves (e-folding decay times of the order of decades are typical). Both t_{ob} and the recurrence time of the outbursts (t_{rec}) remain poorly known as no stars have yet been observed to return to their pre-outburst states. The peak outburst accretion rate is $\sim 10^{-4} M_\odot \text{yr}^{-1}$, which is found by matching numerical simulations to outburst spectra, e.g. Kenyon, Hartmann & Hewett (1988). The mean accretion rate during outburst is $\sim 10^{-5} M_\odot \text{yr}^{-1}$. For a detailed review of the FU Ori and T Tau stars, and references, see Hartmann (1998); Hartmann & Kenyon (1996).

The outbursts of YSOs are not yet fully understood, but are believed to be associated with a thermal-viscous accretion disc instability. Models based on the thermal-viscous disc instability have widely been used as an explanation for similar outbursts in other stellar systems, especially binary stars such as dwarf novae, e.g. Warner (1995). However the long recurrence times of YSOs do not correspond with those predicted by the standard thermal-viscous disc instability model (DIM) with an accretion disc viscosity of the same order as that applied to binaries. In this paper we argue that the standard DIM, with a quiescent Shakura-Sunayev viscosity parameter of $\alpha_c \gtrsim 10^{-3}$, in the case of an accretion disc with an inner radius comparable to the radius of the central star would predict a series of shorter, more frequent outbursts than are observed in any YSOs. One possibility is that the observed outburst behaviour could be reproduced simply by decreasing the Shakura-Sunayev viscosity param-

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eter to a very low value e.g. Bell & Lin (1994). We propose an alternative model which incorporates the effect of a stellar magnetic field on the inner accretion disc. A magnetic field anchored on a rapidly rotating star can act as a propeller (e.g. Lovelace, Romanova & Bisnovatyi-Kogan 1999; Wynn, King & Horne 1997) creating a low density region in the inner disc, and reducing accretion to the order of the CTT rate. Outbursts are thought to begin when the surface density exceeds a critical value $\Sigma_{\text{crit}}^{\text{high}}$ anywhere in the disc. Since the critical density increases with distance from the star (Cannizzo, Shafter & Wheeler 1988), a truncated accretion disc is able to accommodate more mass, forcing outbursts to be more dramatic and much less frequent.

In Section 2 we review estimates of the recurrence and rise times of outbursts in T Tau systems. These estimates are compared with those expected from the standard DIM, we also outline the effect of viscosity. An extension of the DIM, including a magnetic propeller, is discussed in Section 3. An expression for the evolution of the disc in the radial dimension in the presence of a magnetic field is derived in Section 4 and an analytic solution is obtained for the steady state. Finally numerical calculations are performed which demonstrate the feasibility of using a magnetic propeller to create the required depleted region in the centre of a T Tau disc.

2 OUTBURST TIMESCALES

2.1 Disc instability model

The thermal-viscous disc instability model is now generally accepted as the mechanism behind outbursts in, for example, dwarf novae, e.g. (Buat-Ménard, Hameury & Lasota, 2001; Warner, 1995), and other accreting binary systems. However The standard DIM is unable to explain the long timescales associated with the outburst cycle of FU Ori stars at least for similar viscosities to those applied to discs in binaries.

In the thermal-viscous instability model a limit-cycle in temperature and density forces the accretion disc to cycle between two stable states: a hot, high accretion, high viscosity state; and a cold, low accretion, low viscosity state. Once an annulus of a disc reaches the hot state it triggers a heating wave leading to an outburst which affects the whole disc, or at least a portion of the disc which extends to some maximum radius. The DIM is discussed in detail by e.g. Frank, King & Raine (2002); Cannizzo, Shafter & Wheeler (1988). The trigger causing the change from the cold to the hot state can be expressed in terms of a critical value of the disc surface density Σ . Following Cannizzo et al. (1988) we approximate this critical Σ with the formula

$$\Sigma_{\text{crit}}^{\text{high}} = 11.4 R_{10}^{1.05} M_1^{-0.35} \alpha_c^{-0.86} \text{ g cm}^{-2}, \quad (1)$$

where R_{10} is the radius from the star in units of 10^{10} cm, M_1 is the mass of the star in solar masses, and α_c is the Shakura-Sunyaev alpha viscosity during the cold (quiescent) state. It is clear that the lowest $\Sigma_{\text{crit}}^{\text{high}}$, which would trigger outbursts, must appear close to the inner edge of the disc (R_i) and that outbursts would begin near R_i unless some process were to cause a distortion of the disc's surface density profile. The radius at which the outburst is initiated has a strong effect on both t_{ob} and t_{rec} .

It is possible to estimate the outburst duration t_{ob} by considering the e-folding decay time t_e of the observed light curves of FU Ori stars. These timescales are typically in the range of $10\text{yr} \lesssim t_e \lesssim 50\text{yr}$. An FU Ori star in outburst is about two magnitudes brighter than in quiescence, e.g. Hartmann (1998). If the outburst is considered to be over when the star returns to its quiescent luminosity then as a first approximation we may say that the outburst time is of order $t_{\text{ob}} \sim 0.4 t_e \Delta m \ln 10 \lesssim 10^2$ yr, where Δm is the change in magnitude between outburst and quiescence.

Since no FU Ori stars have been observed through a complete outburst cycle it is difficult to estimate t_{rec} from observational data. However it is possible to obtain a rough estimate of t_{rec} by comparing mean mass transfer rates in quiescence, $\langle \dot{M}_{\text{qu}} \rangle$, and during outburst, $\langle \dot{M}_{\text{ob}} \rangle$. Assuming that a large proportion of the region of the disc involved in the outburst is accreted, this mass must be replaced in a time t_{rec} and hence

$$\langle \dot{M}_{\text{ob}} \rangle t_{\text{ob}} \sim \langle \dot{M}_{\text{qu}} \rangle t_{\text{rec}}. \quad (2)$$

Solving relation (2) yields $t_{\text{rec}} \sim 5000$ yr, where we have adopted $\langle \dot{M}_{\text{qu}} \rangle \sim 10^{-7} \text{ M}_{\odot} \text{ yr}^{-1}$ and $\langle \dot{M}_{\text{ob}} \rangle \sim 10^{-5} \text{ M}_{\odot} \text{ yr}^{-1}$ from Hartmann (1998) and the estimate $t_{\text{ob}} \sim 50$ yr. This estimate is likely to be an upper limit as the mass transfer rate through the disc in quiescence must be larger than the observed accretion rate to allow an accumulation of mass leading to the next outburst.

A further approximate analytic limit for t_{rec} in the DIM can be obtained by considering the mass flux through the annulus in the disc at which the outburst is initiated. The surface density of this region must reach $\Sigma_{\text{crit}}^{\text{high}}$ in a time t_{rec} , leading to the relation

$$t_{\text{rec}} \sim \frac{M(R_i)}{\dot{M}(R_i)} \sim \frac{2\pi R_i \Sigma \Delta R}{\langle \dot{M}_{\text{qu}} \rangle}, \quad (3)$$

where $M(R_i)$ is the mass contained in the annulus, which begins close to the inner radius R_i of the disc and extends outwards a distance ΔR . Approximating $\Delta R \sim 0.1 R$ (which is not unreasonable if an aspect ratio of $H/R \sim 0.1$ is adopted throughout the disc) it is possible to express t_{rec} in the form,

$$t_{\text{rec}} \sim \frac{0.2\pi R_i^2 \Sigma}{\langle \dot{M}_{\text{qu}} \rangle}. \quad (4)$$

During quiescence the surface density in the disc is limited by $\Sigma < \Sigma_{\text{crit}}^{\text{high}}$. Therefore equations (4) and (1) can be combined to give

$$t_{\text{rec}} \lesssim 2 \times 10^{-2} M_1^{-0.35} R_{i,10}^{3.05} \langle \dot{M}_{\text{qu},15} \rangle^{-1} \alpha_c^{-0.86} \text{ yr}, \quad (5)$$

where $R_{i,10}$ represents the inner radius of the disc in units of 10^{10} cm, $\langle \dot{M}_{\text{qu},15} \rangle$ is the mean quiescent accretion rate in units of 10^{15} gs^{-1} . In this paper we adopt a value of $\alpha_c \sim 0.01$ which is the quiescent viscosity parameter commonly applied to dwarf novae accretion discs. This assumption will be discussed further in section 2.2. The estimate of the surface density profile that is used here is of course an upper limit, it will be seen however in Section 4 that this is a reasonable approximation. The estimate (5) can be tested by comparing its predictions to the outburst behaviour of the dwarf novae, which have been observed through many outburst cycles. In a dwarf nova the accreting star is a white dwarf with a mass $M_{\star} \sim 1 \text{ M}_{\odot}$, and radius $R_{\star} \sim 0.05 \text{ R}_{\odot}$, and the quiescent accretion rate is

$\langle \dot{M}_{\text{qu}} \rangle \lesssim 10^{-11} \text{ M}_{\odot} \text{ yr}^{-1}$ (Warner 1995). Using these values, and assuming that $R_i \sim R_*$, equation (5) gives an estimate for the recurrence time of $t_{\text{rec}} \sim 10 \text{ d}$, very close to the observed t_{rec} in these systems. In order to apply (5) to a typical FU Ori system we adopt values of $M_* \sim 1 \text{ M}_{\odot}$, and $\langle \dot{M}_{\text{qu}} \rangle \sim 10^{-7} \text{ M}_{\odot} \text{ yr}^{-1}$. If the disc is assumed to extend down to a typical CTT stellar radius of $R_i \sim 2R_{\odot}$, then equation (5) yields a recurrence time of $t_{\text{rec}} \sim 0.5 \text{ yr}$. This is clearly far too short to be consistent with observation. Indeed, to obtain $t_{\text{rec}} \sim 5000 \text{ yr}$ we require an inner disc radius of $R_i \sim 40 R_{\odot}$.

The timescale on which the luminosity of the disc increases (t_{rise}) in an FU Ori outburst is governed by the thermal timescale of the region in which the outburst begins, in the case of an outside-in outburst. Clarke & Syer (1996) have examined this in detail and point out that rise times of $\lesssim 1 \text{ yr}$ imply that outbursts must begin at radii much larger than that of the central star. They estimate values $\gtrsim 15 R_{\odot}$, in rough agreement with the estimates above. If we assume the observed values of t_{rise} are of the same order as the local thermal timescale t_{th} at the point where the outburst begins t_{th} we obtain the relation (Frank et al. 2002)

$$t_{\text{rise}} \sim t_{\text{th}}(R) \sim (\alpha \Omega_k(R))^{-1} \sim \frac{1}{\alpha_c} \left(\frac{R^3}{GM_*} \right)^{1/2}, \quad (6)$$

where $\Omega_k(R)$ represents the Keplerian angular velocity at a given radius. The alpha viscosity parameter in quiescence is once again assumed to be $\alpha \sim 0.01$. The observed rise time of $t_{\text{rise}} \sim 1 \text{ yr}$ is consistent with an inner disc radius of $R_i \sim 40 R_{\odot}$, which is similar to the estimate obtained from t_{rec} and consistent with the large radii predicted by Clarke & Syer (1996).

2.2 Low viscosity models

It has been shown that the above estimates for t_{rec} and t_{rise} are consistent with the DIM if it is modified, for example, by including a truncated inner disc where $R_i \sim 40 R_{\odot}$. We note here however that long values of t_{rec} and t_{ob} , consistent with observation, can be reconciled with the standard DIM if a low value of the quiescent disc viscosity is invoked. Equation (5) has a dependence on α_c and so reducing α_c to small values can yield a t_{rec} which is arbitrarily long. Indeed such a model has been applied by Bell & Lin (1994) and Bell et al. (1995) who use a one dimensional disc model similar to that described in Section 4. Bell & Lin (1994) simulate full outbursts and include detailed thermodynamics in order to calculate the onset of the thermal instability. This is then associated with an arbitrary increase in the disc viscosity. For most of these simulations $\Sigma = 0$ at the inner boundary and \dot{M} is held constant at the outer boundary. With this model Bell & Lin (1994) are able to obtain recurrence times of $t_{\text{rec}} \sim 1000 \text{ yr}$ using the standard DIM with $\alpha_c \sim 10^{-4}$. Our analytic estimate of t_{rec} from Equation (5) requires an even lower value of $\alpha_c \sim 10^{-6}$ to obtain t_{rec} of this order. This discrepancy is because the analytic estimate developed in 2.1 includes the assumption that the outburst begins at a radius of $R = R_i \sim 2R_{\odot}$ whereas according to the results of Bell & Lin (1994) the outburst begins at $R \sim 10R_{\odot}$. Bell & Lin (1994) also include a feedback mechanism whereby the radius of the disc is a function of \dot{M} .

We note here that this model is successful, providing that outbursts can be initiated at $R \gtrsim 10 R_{\odot}$. Of course a similarly long t_{rec} could be replicated by outbursts initiated at smaller radii assuming even lower viscosities. Nonetheless Bell & Lin (1994) do confirm that the dwarf nova like estimate of $\alpha_c \sim 10^{-2}$ is far too high to reproduce values of t_{rec} in agreement with observational limits in the standard DIM.

Theoretical work has been done which suggests that a very low value of α_c is possible in protostellar discs, because the region in which thermal ionisation is able to occur throughout the thickness of the disc is limited to $R \lesssim 0.1 \text{ AU}$ (Gammie 1996). However the FU Orionis outbursts are driven in the inner disc, which is likely to be thermally ionised, and where as a result the viscosity is likely to be similar to that in dwarf novae. Hartmann et al. (1998) have argued that the radii of circumstellar discs can be used to constrain the quiescent viscosity parameter to $\alpha_c \sim 0.01$, similar value to the dwarf nova case, throughout the disc. The remainder of this paper will be dedicated to the investigation of an alternative mechanism which can extend recurrence times, by disc truncation, to within the observational limits while using a dwarf nova like low state viscosity of $\alpha_c \sim 0.01$ in the inner disc.

3 DISC TRUNCATION BY A MAGNETIC FIELD

In Section 2 we argued that the standard thermal-viscous instability outburst model is insufficient to explain the outbursts of the FU Ori stars. A truncated disc however produces reasonable estimates of t_{rec} and t_{rise} . One possible mechanism for truncating the disc is a magnetic field associated with the accreting star.

The suggestion that the accretion disc in YSOs is truncated by the magnetic field of the young star, or otherwise, is not new and has appeared in the literature many times. Examples include Ulrich, Wynn & Regev (2001), who model the CTT infra-red excess with a truncated disc, Armitage & Clarke (1996) who explain T Tau spin regulation with a truncated disc model, and Clarke & Syer (1996). A magnetic mechanism which may lead to disc truncation is investigated below.

3.1 The magnetic propeller

The magnetic fields of the T Tau and FU Ori stars are widely believed to play an important role in their evolution and behaviour, e.g. Yi & Kenyon (1997); Safer (1999). Good estimates of the magnitudes of these fields are, however, difficult to obtain. The most recent estimates available of the magnetic field of the CTT star BP Tau, measured using spectropolarimetry (Johns-Krull et al. 1999), put the value at $B = 2.4 \pm 0.1 \text{ kG}$. This is in good agreement with earlier measurements for the magnetic fields of both CTTs and FU Ori stars, which have been calculated using Zeeman broadening effects, e.g. Johnstone & Penston (1987). The inner accretion disc in a CTT will be partially ionised, as it is irradiated by the accreting star, and will therefore interact with the stellar magnetic field. Angular momentum can be transferred from the disc to the star and vice-versa,

via this magnetic interaction. The magnetic field is therefore important in determining the spin rates of these stars as discussed in more detail in Popham (1996); Armitage & Clarke (1996).

The magnetic propeller effect (e.g. Wynn et al. 1997; Lovelace et al. 1999) involves a net transfer of angular momentum from the star to the disc via the magnetic field. In this case the interaction between the disc and the field takes place at radii larger than the co-rotation radius R_{co} , the point at which the circular Keplerian angular frequency Ω_K of the disc matches the angular frequency Ω_* of the star. For the purposes of this paper we assume that the stellar field is dipolar and that it rotates at the same rate as the star such that

$$\Omega_K(R_{\text{co}}) = \Omega_* = \Omega_B, \quad (7)$$

where Ω_B represents the angular frequency of the magnetic field. At R_{co} it is clear that a body in a circular Keplerian orbit remains stationary in the frame of reference of the magnetic field, and there is no torque between the disc and the field at this point. Hence, the magnetic timescale t_{mag} (the timescale on which magnetic torques redistribute angular momentum in the disc) is infinite at R_{co} . Outside R_{co} we have $\Omega_K < \Omega_*$ and the disc gains angular momentum at the expense of the star. Matter which is accelerated in this manner will be pushed out to greater radii until the magnetic torque (which for a dipole field decreases as $\sim R^{-6}$) becomes small when compared to viscous stresses in the disc. This occurs when $t_{\text{mag}} \gg t_{\text{visc}}$, where t_{visc} is the timescale on which viscous stresses redistribute angular momentum within the disc. The radius at which $t_{\text{mag}} \sim t_{\text{visc}}$ is usually termed the magnetic or magnetospheric radius R_{mag} . Inside R_{co} , the magnetic torque extracts angular momentum from the disc. In this case the disc mass is rapidly accreted onto the star.

The results of both the propeller and the accretion regimes are the formation of a depleted region in the disc close to the star. The radius of this region is $R_i \sim R_{\text{mag}}$, and therefore is a function of B , the spin period P_{spin} of the star, and the disc viscosity. If we assume, as argued in Section 2, that an inner disc radius $R_i \sim 40 R_{\odot}$ is required to be consistent with observational constraints then R_i must exceed R_{co} even for slow spinning T Taus. Although both the accretion and propeller regimes lead to the formation of a depleted region in the centre of the disc, only the propeller mechanism leads to the accumulation of disc mass at radii greater than R_{co} .

Below we derive an expression for the magnetic timescale t_{mag} in a completely magnetised accretion disc in the presence of a dipolar magnetic field. The result is almost identical to that adopted by Livio & Pringle (1992). The inclusion of a magnetic field in a hydrodynamic system introduces two additional terms to the Euler equation; a magnetic pressure term and a magnetic tension term, e.g. Dendy (1990). The magnetic pressure term is negligible where B is small. The magnetic tension term can be expressed as

$$a_{\text{mag}} \sim \frac{1}{\rho_d r_c} \left(\frac{B_z B_\varphi}{4\pi} \right) \hat{\mathbf{v}}, \quad (8)$$

where ρ_d represents the density of the disc, r_c is the local radius of curvature of the field lines and B_z and B_φ represent the vertical and azimuthal components of the magnetic

field respectively. We use the approximations $r_c \sim HB_z/B_\varphi$ (Pearson, Wynn & King 1997), and $H \sim 0.1R$. The ratio of field strengths can be expressed in the form (e.g. Livio & Pringle 1992)

$$\frac{B_\varphi}{B_z} \sim - \frac{(\Omega_k - \Omega_*)}{\Omega_k}. \quad (9)$$

The magnetic timescale can be defined in terms of the magnetic acceleration and the Kepler velocity by the relation

$$t_{\text{mag}} \sim \frac{R\Omega_k}{a_{\text{mag}}} \sim - \frac{2\pi R^2 \rho_d}{5B_z^2} \frac{\Omega_k^2}{(\Omega_k - \Omega_*)}. \quad (10)$$

Finally the volume density can be related to the surface density by the expression $\rho_d \sim \Sigma/H \sim 10\Sigma/R$ and, for a dipole field, we have $B_z \sim \mu R^{-3}$ where μ is the magnetic moment of the star. These relations lead to the following expression for the magnetic timescale

$$t_{\text{mag}} \sim \frac{4\pi\Sigma\sqrt{GM_*}R^{11/2}}{\mu^2 \left(\left[\frac{R}{R_{\text{co}}} \right]^{3/2} - 1 \right)}. \quad (11)$$

It is of course possible to use other prescriptions for the magnetic timescale, for example the diamagnetic case as used by e.g. Wynn et al. (1997). In general however t_{mag} can normally be parametrized in the form

$$t_{\text{mag}} \sim \frac{2\Sigma}{\beta} \frac{R^{(\gamma+2)}}{\left(\left[\frac{R}{R_{\text{co}}} \right]^{3/2} - 1 \right)}, \quad (12)$$

where γ and β are parameters determined by the magnetic interaction model. We use this general expression to derive equation (25) in Section 4. In the fully magnetised case represented by equation (11) we have $\gamma = 7/2$ and β is determined by the mass and magnetic moment of the star according to

$$\beta = \frac{\mu^2}{2\pi\sqrt{GM_*}}. \quad (13)$$

3.2 Magnetic models for CTTs and WTTs

Several authors have considered magnetic truncation of the accretion disc in YSOs. Kenyon, Insu & Hartmann (1996) use this idea to model the infrared excesses of CTTs. Armitage & Clarke (1996) model a magnetically truncated accretion disc and use this model to explain the regulation of the spin periods of T Tau stars on a timescale $\sim 10^5$ yr. The magnetic propeller effect has previously been applied to the case of T Tau accretion discs by Ultchin et al. (2001), who use a smoothed particle hydrodynamics technique to show that a central hole is formed by the magnetic field, exactly as described above and as will be confirmed by numerical results in Section 4. In the full three dimensional case some accretion should occur, as there will be a component of the orbital velocity which is not in the azimuthal direction with respect to the dipole field. The accretion stream follows the magnetic field lines of the accreting star. Indeed Gullbring et al. (1998) find that CTT spectra agree with an accretion stream of this sort better than they do with standard boundary layer accretion. This is analogous, in binary systems, to the flow in the case of the intermediate polar (IP) stars as discussed for example in Murray et al. (1999).

Livio & Pringle (1992) discuss the effect of a magnetic propeller on the onset of outbursts in dwarf novae.

Magnetic models have also been used to explain the transition between CTT and WTT states by allowing the magnetospheric radius R_{mag} to move, which affects the accretion behaviour of the system. When R_{mag} is large, the disc is propelled outwards to large radii and very little accretion takes place. This state can be identified with a WTT system since there would be a low accretion rate and no inner disc. If the star is in a state where R_{mag} is smaller, then it may be that $R_{\text{mag}} < R_{\text{co}}$, in which case the system would become a magnetic accretor. As an alternative to the magnetic accretor, a weak propeller could be invoked. In this case $R_{\text{mag}} > R_{\text{co}}$ but R_{mag} is still smaller than the strong field value. In either case these systems could then be identified as CTTs as a disc would exist close to the star, although not extending to the surface. This disc could produce the observed infra-red excess and emission line characteristics of CTTs.

Clarke et al. (1995) suggest a ‘magnetic gate’ where magnetic cycles in the star cause a change in the magnetospheric radius, R_{mag} . A second possibility is that proposed by Armitage & Clarke (1996) where R_{mag} can be changed by spinning the star up or down by disc interactions. High spin stars, with a greater R_{mag} , would then be classified as WTTs, which is in agreement with observation. A rapid spin up can most simply be achieved by a high accretion phase such as an FU Ori outburst. The gradual spin down of the system in its WTT phase and transition to CTT is modelled in detail by Armitage & Clarke (1996). The effects of a magnetically truncated disc on the spin evolution of the T Tau stars will be considered in Section 3.4.

3.3 A magnetic model for FU Orionis outbursts

As discussed in Section 3.2, the action of a magnetic propeller in the disc of a T Tau object will cause the truncation of the disc at some point outside R_{co} . The precise position of truncation depends on the strength of the stellar magnetic field, the rate of spin of the star and the viscosity of the disc. If a depleted region, or hole, is formed then the total mass \dot{M}_{ob} of the disc is allowed to reach a higher value before $\Sigma_{\text{crit}}^{\text{high}}(R_i)$ is exceeded and an outburst begins, increasing both t_{ob} and t_{rec} . During the quiescent, or propelling, phase mass transfer is not completely prevented, but is reduced to the CTT accretion rate of $\dot{M} \sim 10^{-7} \text{ M}_{\odot} \text{ yr}^{-1}$. Some accretion will still occur because in three dimensions, as mentioned in Section 3.2, some component of the azimuthal velocity is not perpendicular to the magnetic field. The mass which leaks through the magnetic barrier accretes along the magnetic field lines onto the central star, which is analogous to the behaviour of other magnetic propeller systems. In a forthcoming paper we attempt to apply a similar magnetic mechanism to the cataclysmic variable WZ Sagittae, in which case the quiescent accretion rate is ~ 1 percent of the outburst rate. Indeed even in the interacting binary AE Aquarii (Wynn et al. 1997), a propeller system so strong that it completely prevents the formation of a disc, the accretion rate is ~ 1 percent of secondary mass loss rate. It should be noted that this accretion is an intrinsically three dimensional effect and that the situation is somewhat differ-

ent in the one dimensional model applied later in the paper. This distinction is discussed in section 4.1.

It is of course possible to imagine a disc in which the propeller is sufficiently strong to prevent outbursts. In this case a steady state may exist in which the accretion rate is constant and the critical density is never reached. The parameters for which this is possible will be investigated in section 4.2. It should be stressed here however that in order for any steady state to exist, some ‘leakage’ of material through the magnetic barrier is required. The relative frequency of outbursting and non-outbursting CTTs will depend on the distribution of magnetic moments and of disc viscosities. The discussion of this effect requires a full parameter search and is left to future work.

If the disc goes into outburst, the viscosity is believed to increase by up to an order of magnitude. The resulting decrease in t_{visc} reduces R_{mag} to a point much closer to, or even within, the surface of the star, and causes \dot{M} to increase by several orders of magnitude to the peak FU Ori accretion rate of $\dot{M} \sim 10^{-4} \text{ M}_{\odot} \text{ yr}^{-1}$. The dissipative (viscous) effect dominates over the advective (magnetic) effect at all radii in this case. This continues to be the case until much of the stored mass in the disc has been accreted and the disc is allowed to return to the cold state. Indeed for as long as the condition $R_i < R_{\text{co}}$ is satisfied, the magnetic field will actually assist accretion in the inner disc.

On the disc returning to the cold state, R_i is forced out once more to its original position outside R_{co} , assuming that neither B nor P_{spin} are significantly altered by the outburst. The system should now return to its pre-outburst state, with a CTT accretion rate, and the mass of the outbursting region should begin to build up once more. Hence, providing that a magnetic propeller can produce a depleted region, it can be invoked to explain the long t_{rec} as well as the long t_{rise} in FU Ori systems.

3.4 Spin evolution

Since P_{spin} effects R_{mag} and hence has an influence on t_{rec} , it is important to consider the effect of spin evolution on FU Ori outbursts. This can be done to a first approximation by comparing the spin up timescale of the star to t_{rec} and t_{ob} .

According to Armitage & Clarke (1996) the squared radius of gyration of a T Tau star is $k^2 \sim 0.2$ for most of its lifetime. We adopt this value of k^2 and assume a solar mass star with a typical T Tau spin period of $P_{\text{spin}} \sim 3 \text{ d}$. The rotational angular momentum of the star is given by

$$L_{\star} = k^2 M_{\star} R_{\star}^2 \Omega_{\star} . \quad (14)$$

Away from the inner radius angular momentum transport in the accretion disc is dominated by viscous effects. It is only at the inner edge of the disc that magnetic effects become important.

During quiescence the magnetic propeller will cause the star to experience a net spin down torque. Armitage & Clarke (1996) suggest that a propeller will spin down the star on a timescale of $t_{\text{spin,qu}} \sim 10^5 \text{ yr}$. It is likely then that stellar spin evolution is insignificant during a single quiescent phase.

In the case of magnetic accretion the spin up torque on

the star is given by $\dot{L}_\star = \dot{M}l(R_i)$, where $l(R_i)$ represents the specific angular momentum of matter at R_i , giving

$$\dot{L}_\star = \dot{M}\sqrt{GM_\star}R_i^{1/2}. \quad (15)$$

The spin up timescale can be expressed as

$$t_{\text{spin}} \sim L_\star / \dot{L}_\star. \quad (16)$$

During outburst $\langle \dot{M}_{\text{ob}} \rangle \sim 1 \times 10^{-5} \text{ M}_\odot \text{ yr}^{-1}$ and $R_i \sim R_\star \sim 2 R_\odot$ because the central region is likely to have become filled. From equation (16), this yields $t_{\text{spin,ob}} \sim 10^3 \text{ yr}$. Therefore even in outburst $t_{\text{spin,ob}} > t_{\text{ob}}$ compared with an estimated $t_{\text{ob}} \sim 10^2 \text{ yr}$. In this case it is sensible to assume that the measured \dot{M} is the same as that at the surface of the star and to neglect transport of angular momentum from the star to the disc. This result shows that spin evolution should therefore not have a significant influence on outburst behaviour during one outburst, although there may be a measurable cumulative effect.

Although spin evolution may turn out to be significant in the longer term for T Tau stars, it is still possible to illustrate the principle of outburst and recurrence time behaviour with a fixed spin and hence a fixed R_{mag} . It is assumed for the purposes of this paper that the outbursts occur in the CTT phase and that the system returns directly to a similar CTT state after outburst. Any magnetic evolution of the T Tau star is neglected.

4 THE STRUCTURE OF A TRUNCATED DISC

4.1 Analytic Solution

A thin, axisymmetric accretion disc is assumed to be rotating about a central mass M_\star . As we are only concerned with the inner Sections of the disc where outbursts begin, and the total disc mass can be estimated to be $\sim 10^{-3} \text{ M}_\odot$ (Hartmann 1998), it is reasonable to neglect self-gravity in this portion of the disc. Averaging the basic hydrodynamic equations over all azimuthal phases and integrating over the direction normal to the plane of the disc, the continuity equation becomes

$$\frac{\partial \Sigma}{\partial t} + \frac{1}{R} \frac{\partial (R \Sigma v_R)}{\partial R} = 0, \quad (17)$$

the radial component of the Navier Stokes equation becomes

$$\Sigma \left(\frac{\partial v_R}{\partial t} + v_R \frac{\partial v_R}{\partial R} - \frac{v_\varphi^2}{R} \right) = - \frac{\partial p}{\partial R} - \Sigma \frac{GM}{R^2} + \frac{4}{3R^{3/2}} \frac{\partial}{\partial R} \left[R^{3/2} \nu \Sigma \frac{\partial v_R}{\partial R} \right] - \frac{2}{3R^3} \frac{\partial (R^2 \nu \Sigma v_R)}{\partial R}, \quad (18)$$

and the azimuthal component of the Navier Stokes equation becomes

$$\Sigma \left(\frac{\partial v_\varphi}{\partial t} + \frac{v_R}{R} \frac{\partial (R v_\varphi)}{\partial R} \right) = \frac{1}{R^2} \frac{\partial}{\partial R} \left[R^3 \nu \Sigma \frac{\partial}{\partial R} \left(\frac{v_\varphi}{R} \right) \right], \quad (19)$$

where v_φ and v_R denote the azimuthal and radial components of the velocity respectively and ν represents the kinematic viscosity of the disc.

By multiplying the azimuthal component (19) of the equation of motion with R , we obtain an equation for the specific angular momentum in the disc,

$$\Sigma \left(\frac{\partial l}{\partial t} + v_R \frac{\partial l}{\partial R} \right) = \frac{1}{R} \frac{\partial}{\partial R} \left[R^3 \nu \Sigma \frac{\partial}{\partial R} \left(\frac{l}{R^2} \right) \right] + \Sigma \Lambda. \quad (20)$$

Here, we have already assumed an additional external torque acting on the disc, where Λ is the injection rate of angular momentum per unit mass, following the same procedure as Lin & Papaloizou (1986). A similar procedure is also employed by Pringle (1991) in the case of a binary system.

Assuming a sufficiently cold disc and therefore that the dynamical timescale $t_{\text{dyn}} \sim R/v_\varphi$ is much smaller than any other timescale, we can adopt a Keplerian approximation for the azimuthal motion such that $v_\varphi = (GM/R)^{1/2}$. Then, (20) can be solved for the radial velocity in the disc,

$$v_R = - \frac{3}{R^{1/2} \Sigma} \frac{\partial (R^{1/2} \nu \Sigma)}{\partial R} + 2\Lambda \frac{R^{1/2}}{\sqrt{GM}}. \quad (21)$$

Inserting equation (21) into the continuity equation (17) results in an evolution equation for the surface density of the disc,

$$\frac{\partial \Sigma}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left[R^{1/2} \frac{\partial (R^{1/2} \nu \Sigma)}{\partial R} \right] - \frac{1}{R} \frac{\partial}{\partial R} \left[2\Lambda \Sigma \frac{R^{3/2}}{\sqrt{GM}} \right], \quad (22)$$

where the right hand side is composed of a diffusion term on the left and an advection, or torque term on the right.

To parametrize the specific torque Λ , we write

$$\Lambda = \frac{l}{t_\Lambda} = \frac{\sqrt{GM} R^{1/2}}{t_\Lambda}, \quad (23)$$

where t_Λ is the timescale on which the local disc material gains angular momentum.

In the present case, the source of the torque is the magnetic interaction of a rapidly rotating, moderately magnetic YSO within the disc. The torque timescale is therefore equivalent to the magnetic timescale, $t_\Lambda \sim t_{\text{mag}}$. This magnetic timescale is given in equation (12).

In general, the kinematic viscosity ν may depend on radius as well as on surface density, $\nu = \nu(R, \Sigma)$. For the purposes of this paper ν is taken to be independent of Σ and parametrized according to a power law such that $\nu = \delta R^\epsilon$. Kinematic viscosity may also change with time according to an outburst limit cycle. It may be possible to implement this form of ν and, with suitable boundary conditions, to simulate complete outburst cycles in the future. The viscous timescale is given by (Frank et al. 2002)

$$t_{\text{visc}} \sim \frac{R^2}{\nu} \sim \frac{R^2}{\delta R^\epsilon}. \quad (24)$$

By considering equation (12), a general expression for the advection term in equation (22) is obtained

$$\frac{1}{R} \frac{\partial}{\partial R} \left[2\Lambda \Sigma \frac{R^{3/2}}{\sqrt{GM}} \right] = \frac{\beta}{R} \frac{\partial}{\partial R} \left[\frac{1}{R^\gamma} \left(\left[\frac{R}{R_{\text{co}}} \right]^{3/2} - 1 \right) \right]. \quad (25)$$

Equation (22) can be solved analytically for the steady state. By definition in the steady state $\dot{M} = -2\pi R \Sigma v_R$ is a constant throughout the disc. Using equation (21) and with a magnetic torque as defined in equation (23) this condition can be used to derive a general solution for Σ in steady state,

$$\Sigma = \frac{\dot{M}}{3\pi\delta} R^{-\epsilon} + \frac{\beta R^{-(\gamma+\epsilon)}}{3(2-\gamma)\delta} \left(\left[\frac{R}{R_{\text{co}}} \right]^{3/2} - \frac{(2-\gamma)}{(\frac{1}{2}-\gamma)} \right) + \frac{C}{\delta} R^{-(\frac{1}{2}+\epsilon)}, \quad (26)$$

where C is an arbitrary constant. As will be seen this equation has a form such that Σ becomes negative close to the

star. The inner part of this steady state solution is thus clearly unphysical. It is difficult therefore for a physically motivated inner boundary condition to be chosen. Furthermore the general shape of the function is rather sensitive to this boundary condition. The outflow condition $\partial\Sigma/\partial R = 0$ will be applied however as a first approximation. The inner boundary is taken to be very close to the surface of the star. This gives, in its most general form,

$$C = \frac{\beta R_*^{-\gamma}}{3\left(\frac{1}{2} + \epsilon\right)} \left(\frac{(\gamma + \epsilon)}{\left(\frac{1}{2} - \gamma\right)} R_*^{1/2} + \frac{\left(\frac{3}{2} - \gamma - \epsilon\right)}{(2 - \gamma)} \frac{R_*^2}{R_{co}^{3/2}} \right) - \frac{\epsilon \dot{M} R_*^{1/2}}{3\pi\left(\frac{1}{2} + \epsilon\right)}. \quad (27)$$

This analytic solution for the steady state is precisely reproduced by the numerical solution in the following Section. It is however instructive to examine the analytic form of the solution. It can be seen from the form of equation (26) that, counter-intuitively, in steady state the position of R_i , defined here as the point where $\Sigma = 0$, is independent of δ . The disc structure at very large radii tends to the non-magnetic case as would be expected. The non-magnetic solution is dependent only on \dot{M} , and ν . It is almost identical to the solution for the steady thin disc presented by Frank et al. (2002). Indeed this solution,

$$\nu\Sigma = \frac{\dot{M}}{3\pi} \left(1 - \left[\frac{R_*}{R} \right]^{1/2} \right), \quad (28)$$

can be reproduced precisely by applying an inner boundary condition such that $\Sigma = 0$. In this case, as well as in the full magnetic case it holds that $\Sigma \propto 1/\nu$.

The steady state solution in one dimension cannot represent a physical situation. In steady state \dot{M} is constant throughout the disc by definition. However in the steady state solution found above there is an evacuated region in the disc through which no matter can pass. Numerically this represents the transport of negative mass, but this clearly has no physical meaning. In three dimensions however it is possible to imagine similar behaviour by allowing mass to flow along magnetic field lines in the inner disc. The one dimensional solution can then be taken as an approximation of the behaviour of matter in the plane of the disc. This approach is given some validity by the similarity of these one dimensional results to those obtained in three dimensional simulations. This can be seen in e.g. Uchkin et al. (2001) and also in our own preliminary three dimensional computations.

4.2 Numerical Solutions

It will be shown by solving equation (22) numerically, that it is feasible to create a depleted region of $R_i \sim 40 R_\odot$ in the centre of a T Tau accretion disc with a magnetic propeller and hence to increase t_{rec} to arbitrarily long times. The code is similar to that used by Armitage & Clarke (1996) to model a T Tau disc about a magnetic star, and their results are similar to those presented below. The results are also comparable to those of Pringle (1991). In all cases the simulations tend towards a steady state which can be expressed by the analytical solution derived above. The analytical solution has been deliberately kept in as general a form

as possible, however for the numerical calculations $\epsilon = 3/4$ (Frank et al. 2002) and $\gamma = 7/2$ have been adopted. The computation is sensitive to resolution. Convergence with the analytic solution only occurs at high resolution and when the inner boundary conditions match.

The solver code evaluates the diffusive and advective terms separately and combines them using operator splitting. The diffusive term is computed with a modified Crank-Nicholson scheme (Press et al. 1992) and the resulting values of Σ are used as input for a modified two-step Lax-Wendroff scheme (Press et al. 1992) which is used to solve the advective term.

The first pair of simulations were performed starting from a Gaussian density distribution, using the code described above. This initial distribution represents a viscously spreading ring. One simulation was with, and one without, an advection term, representing a magnetic field. The parameters used in the code were as follows: the mass of the accreting star was taken to be $M_* = 1 M_\odot$. The co-rotation radius was calculated on the basis of a star spinning with a typical T Tau period of 3 days which yields $R_{co} \sim 10 R_\odot$. The boundary conditions applied at both the inner and outer radii of the disc are outflow conditions and the initial density distribution was centred at a radius of $5 R_{co}$. The disc was simulated from $0.1 R_{co} \sim 1 R_\odot$ to $10 R_{co}$. The value $\beta \sim 10^{-13}$ is consistent with an object of $R \sim 1 R_\odot$ and $B \sim 1$ kG.

With the parameters described above, the effect of an advective term on the surface density is illustrated in Figure 1. In the non-magnetic case, where the diffusion term acts alone, the initial Gaussian mass distribution quickly moves towards the accreting star, as expected. This is very similar to the well known result of Pringle (1981). In the magnetic case, the evolution of the outer edge of the modelled region is very similar to the non-magnetic case. This is not surprising considering the $R^{-7/2}$ dependence of the advection term and is in agreement with what was expected from the analytic solution. At the inner edge however, it can clearly be seen that a depleted region of radius $\sim 1.5 R_{co}$ is formed. It is interesting to observe how matter in the initial Gaussian distribution is transported by the diffusion and advection terms. Radial mass transfer rate \dot{M} is plotted in Figure 2. The magnetic and non-magnetic cases appear quite similar upon first inspection. Initially the movement of the mass in the Gaussian is dominated by a diffusion effect. This is represented by the two large lobes. However, in the non-magnetic case the flux near the inner boundary increases in magnitude with time. This represents an accretion process. In the magnetic case there is no accretion, and the inward flux rapidly disappears with time and the outward flux, driven by advection, is dominant at later times.

Further simulations were performed using a constant inflow of $\dot{M} \sim 10^{-7} M_\odot \text{yr}^{-1}$ at the outer boundary and an outflow condition at the inner boundary. In this case an empty disc is assumed at the beginning of the simulation. These conditions are qualitatively a more realistic representation of the formation of the inner disc after an outburst, and should more correctly illustrate where mass build up would first trigger a new outburst. Figure 3 shows the outcome of these simulations with varying magnetic field strengths. The critical density Σ_{crit}^{high} is marked on these

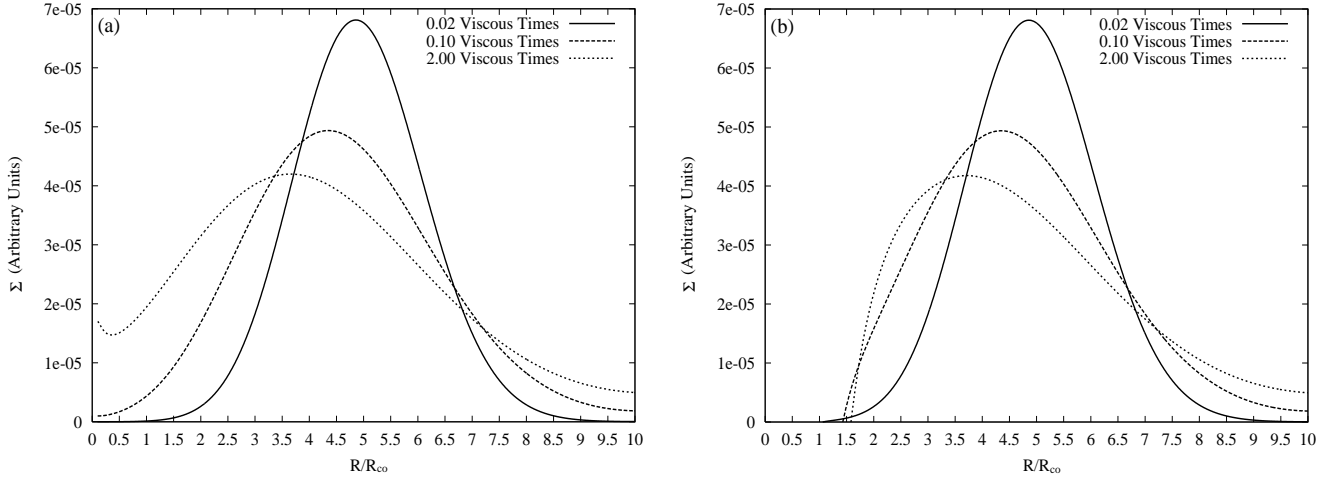


Figure 1. Plot of surface density Σ (in code units) against radius for a spreading Gaussian disc with: (a) no advective term and; (b) a strong advective term which results from a magnetic field. The evolution of the depleted region or ‘hole’ is clearly visible in the latter case.

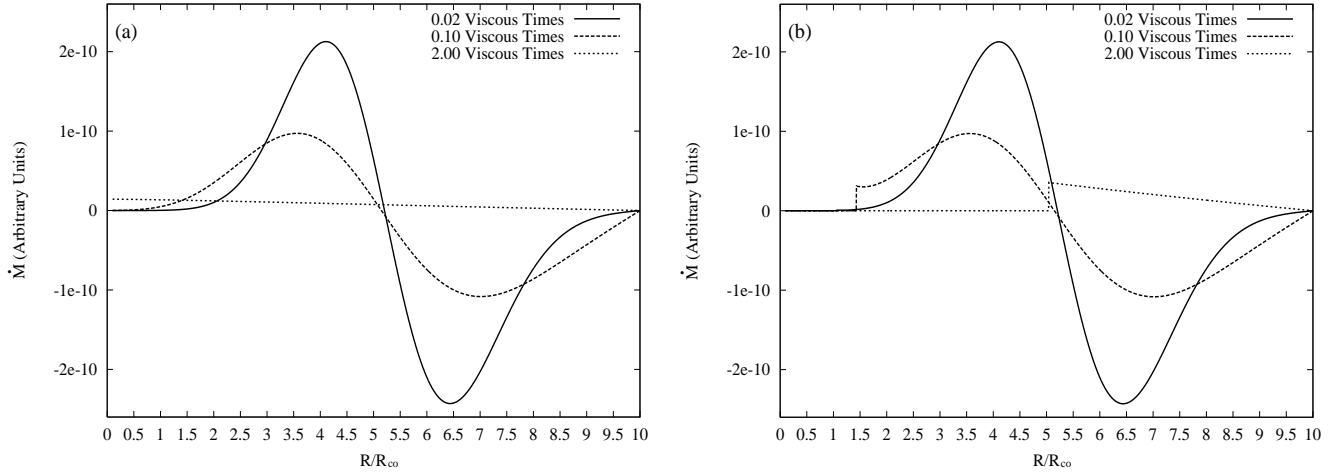


Figure 2. Plot of mass transfer rate, \dot{M} (in code units), against radius for a spreading Gaussian disc with: (a) no advective term and; (b) a strong advective term which results from a magnetic field. Where $\Sigma < 0$, \dot{M} is not plotted. These are the same simulations, plotted at the same times as Figure 1.

graphs to show the position in the disc where surface density first exceeds $\Sigma_{\text{crit}}^{\text{high}}$ and outbursts would begin.

Figure 3(a) represents a simulation of the disc with no magnetic field. The inner radius of the simulation lies at the surface of the star $R_{\star} \sim 1 R_{\odot}$. Plot (a) is on the same scale as plots (b) and (c) for ease of comparison, and plot (a)i is a zoomed in view of graph (a). It can be seen in plot (a) that a fairly flat surface density profile is formed with an upturn in surface density developing towards the surface of the star. The surface density rises with time and the critical density is first reached at the surface of the star, as expected.

Also in Figure 3 the effects of weaker and stronger propellers can be seen in plots (b) and (c) respectively. In both cases the early stages of disc evolution are not dissimilar to those in case (a) for most of the disc. The surface density increases with time and as it does so the effect of the magnetic field becomes apparent. In case (b) the inflow of mass

towards the star is suppressed inside $\sim 2R_{\odot}$ and the critical density is first exceeded at $\sim 4R_{\odot}$. This occurs much later than in plot (a), as expected.

In plot 3(c) a stronger propeller is illustrated, where the advection term is greater. The plot is qualitatively very similar to plot (b) but the completely depleted region extends to $3R_{\odot}$ and the outburst would begin at $\sim 5R_{\odot} \sim 50 R_{\odot}$. Although a full outburst cycle is not simulated and therefore it is not possible to draw detailed conclusions about t_{rec} , it can be seen that outbursts would begin later with increasing β in these simulations. Values of R_i of the expected order of magnitude can be seen in Figure 3. If the outbursting region has a width of order a tenth of the radius at which the outburst begins, as assumed in Section 2, then the approximation that $\Sigma \sim \Sigma_{\text{crit}}$ in this region is clearly a good one in all three cases.

The mass transfer \dot{M} in the disc for all these simulations

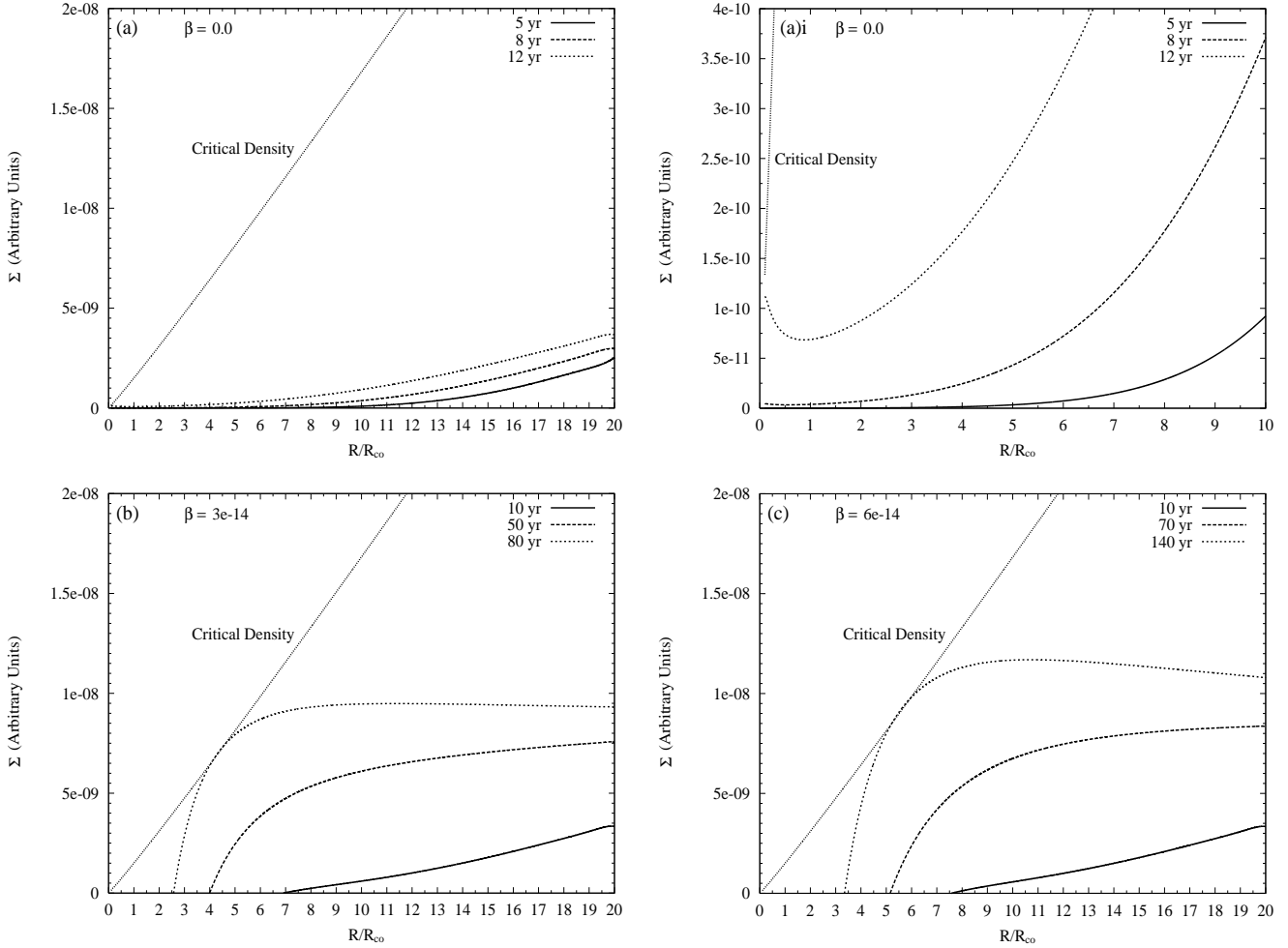


Figure 3. Plots of surface density Σ (in code units) against radius where advective terms of varying strengths, representing magnetic fields, have been applied. In all cases the initial conditions were an empty disc with a constant inflow of $\dot{M} \sim 10^{-7} M_{\odot} \text{yr}^{-1}$ from the right. Plot (a) shows the behaviour of a disc with a zero magnetic field while plots (b) and (c) represent a weak and a stronger field respectively. In both magnetic cases an inner disc truncation can clearly be seen, although this occurs at a greater radius in case (c). Plot (a.i) is a ‘zoomed in’ version of plot (a) showing where the density first exceeds $\Sigma_{\text{crit}}^{\text{high}}$, while plots (a), (b) and (c) are shown on identical axes for ease of comparison. In each case the latest plot is at the point where Σ first reaches the critical density.

is plotted in Figure 4 at the same times as in Figure 3. In all cases the disc has yet to reach steady state at the point where the outburst begins. If no outburst occurred, the disc would eventually approach steady state. Because the spreading of the disc slows as it nears steady state, the longest recurrence times will be observed when the critical density is reached at, or shortly before, steady state. When β is increased so that steady state is reached with the surface density maximum at larger radii then no outbursts will occur and the disc will remain quiescent indefinitely. Such a steady state without outbursts could exist for example, in our results, when $\beta > 1 \times 10^{-13}$. This is equivalent to a stellar magnetic field of $B > 1 \text{ kG}$ in a typical CTT disc with $\alpha_c \sim 0.01$.

It is possible to observe trends in both R_i and t_{rec} as a function of β . Several simulations were performed with different values of the parameter β . As expected, the inner radius increases with β . This increase can be fitted very well with a power law of index $\sim 1/3$. The results are shown in

Figure 5. The values of R_i obtained here are consistent with those which are required in Section 2.

In Figure 6 we plot the relation between β and time until first outburst, starting with an empty disc. In plot (a) the time before outburst increases rapidly beyond a certain β . This is a result of the disc model approaching steady state. This means that it is possible to obtain an arbitrarily long recurrence time provided the star has a sufficiently strong magnetic moment. The value of β at which this occurs is a function of δ but the form remains the same. Plot (b) shows an enlarged section of plot (a). We additionally plot the corresponding inner disc radius. The relation between R_i and the time before first outburst can be fitted by an expression of the form $t \propto R_i^3$ as would be expected from Equation (5). Further, when the time before outburst is long, the inner radius of the disc is close to $R_i \sim 40 R_{\odot}$ as predicted in Section 2. However these results do not provide quantitative estimates of t_{rec} because the initial conditions are so different from those just after an outburst.

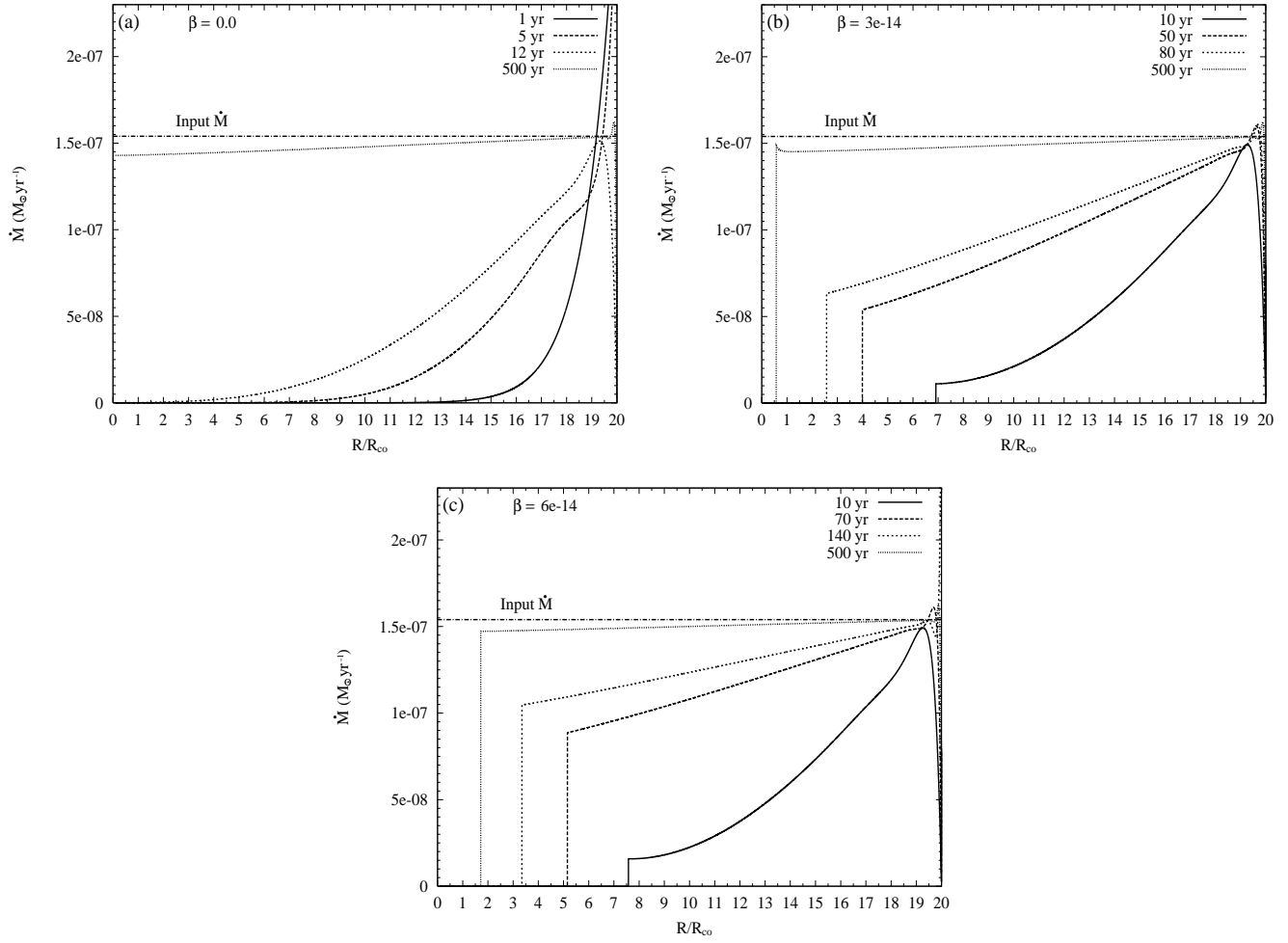


Figure 4. Plots of \dot{M} (in code units) against radius for the same three advective terms strengths as were used in Figure 3. The input \dot{M} is plotted for comparison. Once again, where $\Sigma < 0$, \dot{M} is not plotted. If no outburst occurred then the disc would eventually approach steady state. This can be seen in the plots at 500 yr, where the disc has been allowed to evolve without outburst.

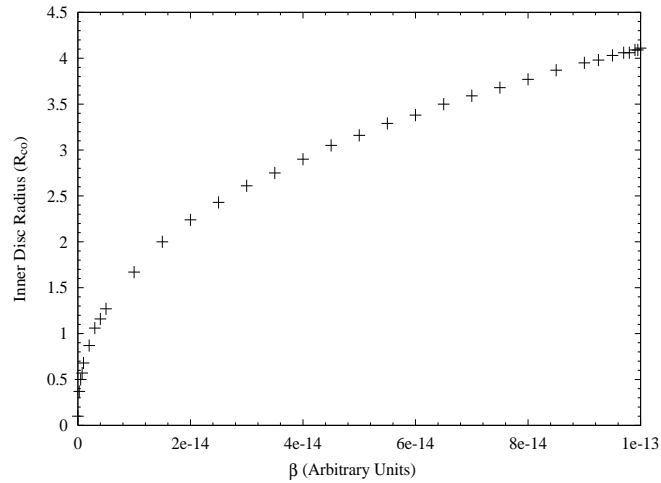


Figure 5. Plot of inner disc radius R_i against magnetic parameter β for a viscosity parameter of $\delta = 2 \times 10^{-8}$ and β in code units. The curve can be fitted well by a power law of the form $R_i \propto \beta^{1/3}$.

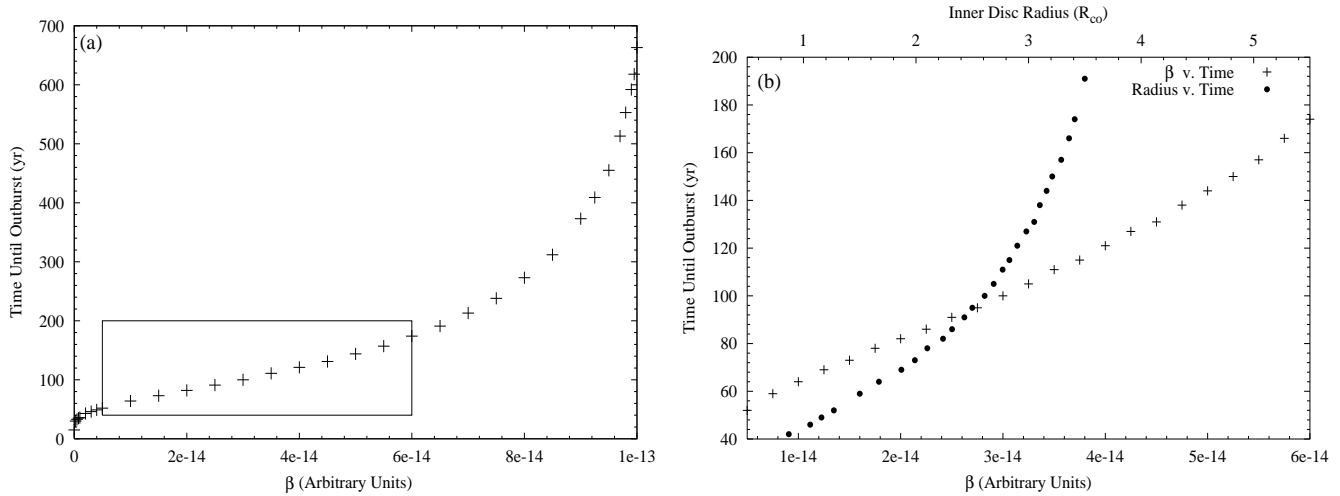


Figure 6. Plots of time before first outburst against magnetic parameter β . In plot (a) the time before the outburst increases very rapidly beyond a certain β . There, steady state is nearly reached before outburst. Plot (b) is a ‘zoomed in’ view of plot (a), also indicating the corresponding inner disc radii.

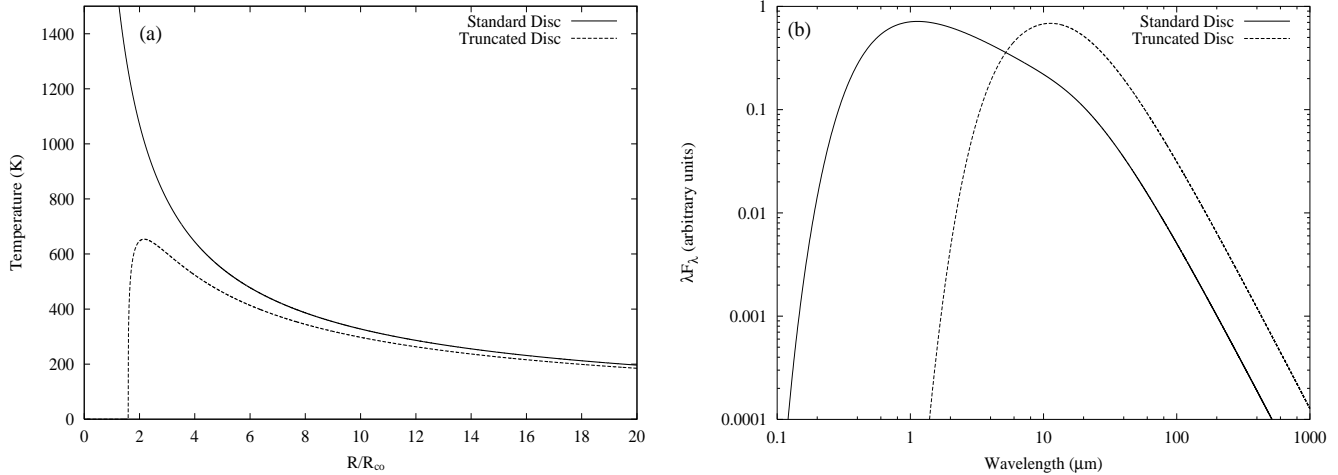


Figure 7. Plot (a) shows temperature in Kelvin as a function of radius in steady state disc models with and without magnetic truncation. The hottest part of the disc is missing in the truncated case. Plot (b) shows the spectral energy distribution in arbitrary units against wavelength in microns. In the truncated case the flux is reduced for wavelengths below $10\mu\text{m}$.

4.3 Simulated spectra

In order to compare the results of numerical models to observations it is useful to produce simulated spectra. This can be done in a simple way by dividing the disc into a series of concentric annuli. The viscous dissipation in each annulus can then be approximated by (Frank et al. 2002)

$$D(R) = \frac{9}{8} \nu \Sigma \frac{GM_{\star}}{R^3}, \quad (29)$$

where ν is the Shakura-Sunyaev viscosity as given by Equation (24) and Σ is taken from simulation results. Stellar irradiation may effect the disc temperature and hence the SED. Following the approach of Adams & Shu (1986) the total luminosity of the disc can be broken down into components due to viscous dissipation and to disc irradiation. The viscous component can be approximated by (Frank et al. 2002)

$$S_v = \frac{GM_{\star} \dot{M}}{2R_{\star}}. \quad (30)$$

In the limit of a large disc, the component of disc luminosity due to irradiation can be approximated by (Adams & Shu 1986)

$$S_i = \pi R_{\star}^2 \sigma T_{\star}^4, \quad (31)$$

where T_{\star} denotes the temperature of the star. The two components can be compared using typical CTT parameters of $T_{\star} \sim 4000$ K, $M_{\star} \sim 1 M_{\odot}$, $\dot{M} \sim 10^{-7} M_{\odot} \text{yr}^{-1}$ and $R_{\star} \sim 1 R_{\odot}$. In this case, the component of disc luminosity due to irradiation is of the order of a few percent of that due to viscosity. It is therefore reasonable to neglect irradiation in the simple model applied here. Considering viscous heating only and assuming that the disc is in thermal equilibrium,

the temperature of each annulus can be calculated from the Stefan-Boltzmann law according to

$$D(R) = e\sigma T^4, \quad (32)$$

where e is emissivity (assumed to be unity), σ is the Stefan-Boltzmann constant and T represents the effective temperature. Temperature profiles, calculated using this simple model, can be seen in Figure 7(a) for both the truncated and non-truncated disc models. A spectrum emitted from each annulus is then generated, assuming that the disc behaves as a black body. The spectral radiance is then given by the Planck radiation law so that

$$R_\nu = \frac{2\pi h\nu^3}{c^2 \left(e^{\frac{h\nu}{kT}} - 1 \right)}, \quad (33)$$

where h represents Planck's constant, ν is the frequency of the radiation, k is Boltzmann's constant and c is the speed of light. All the spectra of the annuli are summed to give the spectral energy distribution (SED) of the entire disc. A result is presented Figure 7(b). In the case of the truncated disc the flux at wavelengths shorter than $1\mu\text{m}$ is greatly reduced. This is to be expected, as the hottest part of the disc has been significantly depleted. This may help to explain the infrared excess observed in some YSOs. The shape of the SED at longer wavelengths is not fully resolved as the contributions of the outer disc, beyond $200 R_\odot$, have not been assessed.

Observed spectral energy distributions of CTTs tend to include an infrared excess which is attributable, at least in part, to the presence of a circumstellar accretion disc (Bertout, Basri & Bouvier 1988; Adams, Emerson & Fuller 1990). Itoh et al. (2003) successfully reproduce observed SEDs for CTTs by summing the SED expected from the young stars themselves with disc SEDs. The position in wavelength of the inner edges, and peaks, of the disc SEDs used by Itoh et al. (2003) lie between those of our truncated and non-truncated models which have been allowed to reach the steady state without reaching outburst. Ulchin et al. (2001) argue that a truncated accretion disc produces an infrared excess more closely in agreement with observations than an unmodified disc model. It should be noted that it is very difficult to separate the disc component from the dominant stellar component of the observed SEDs at wavelengths shorter than $10\mu\text{m}$. Detailed comparison with observation is problematic as the method used to generate disc SEDs from density profiles in this paper is only very approximate, and takes no account of material which is able to reach radii less than R_{mag} . Further comparison of observational results to the predictions of our model would therefore require a more detailed analysis than is possible under the restrictions of a one dimensional treatment.

5 DISCUSSION

The FU Ori stars undergo what appear to be thermal-viscous outbursts. It is very difficult however to reconcile observational estimates of t_{rec} and t_{rise} with the very short estimates obtained from the standard thermal-viscous disc instability model with $\alpha_c \gtrsim 10^{-3}$. It is possible to do so by using a very low value of α_c . We propose an alternative

scheme in which outbursts occurring in an accretion disc truncated at a radius $R_i \sim 40 R_\odot$ are consistent with the observational estimates of the outburst, recurrence, and rise times. The central star, acting as a magnetic propeller, can produce the necessary truncation of the disc. The results of our one dimensional model confirm the feasibility of this idea. A depleted region in the centre of the disc can be made large enough to produce t_{rec} similar to those estimated from observations. In particular, an arbitrarily long t_{rec} can be obtained by selecting appropriate values for viscosity and magnetic field strength.

The simulation of a full outburst in a refinement of the one dimensional code described above, or using a three dimensional technique such as smoothed particle hydrodynamics, would provide more accurate estimates of the outburst timescales, which could be compared to analytical results and to observations. Three dimensional simulations would be particularly valuable as they would produce a self consistent picture of the inner disc. It would also be desirable to treat viscosity in a more rigorous manner. A more extensive study of stellar spin evolution would allow quantitative analysis of any possible feedback mechanisms.

The prolongation of recurrence times in accreting objects as a result of the magnetic propeller mechanism need not be confined to young stellar objects. The same mechanism could potentially exist in some accreting binaries. In YSOs the full significance of the magnetic propeller may not yet be known. It is possible for example that a depleted inner disc may have an effect on the planet formation process.

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